# 16

# *Systemic Risks in Central Counterparty Clearing House Networks*

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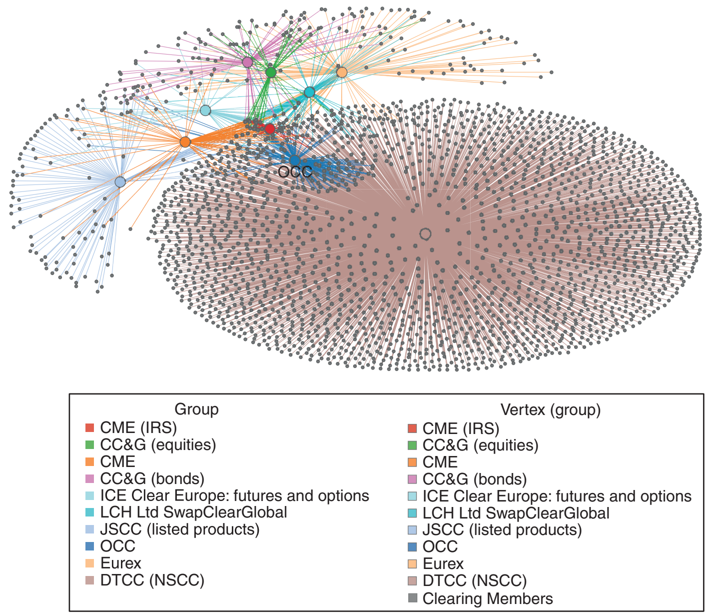
The global financial crisis of 2007–10 had enormous implications for the financial ecosystem as a whole. Among many other changes to its way of working, both the range of products and the number of trades cleared by central counterparty clearing houses (CCPs) increased enormously (see, for example, US Office of Financial Research 2017). As a result, whether they like it or not, all large banks are engaged in trading on CCPs. Accordingly, there is a clear need for banks to assess any potential losses due to defaults of general clearing members (GCMs) and CCPs through the CCP network they participate in. The interconnectedness of the CCPs themselves, arising due to the fact that they are linked through common clearing members, means that it is important to model most of the network.

In this chapter, we take the perspective of a hypothetical banking group, "XYZ Bank", and explain how it may assess its risks based on the partial information available to it. Typically, a banking group has multiple subsidiaries, each of which are distinct clearing members.

Understanding the risk of XYZ Bank is a challenging task, which requires analysing the contingent cashflows between a large number of agents (hundreds of GCMs operating on multiple CCPs) that have a complex interrelationship. To describe this relationship adequately requires capturing the dynamics of variation margin (VM), initial margin (IM), default fund (DF), reassignment of trades in the event of a member default and allocation of these default losses.

**Figure 16.1** A real-world network of CCPs (coloured circles) and GCMs (grey dots) with corresponding edges denoting membership.




*Source:* Barker *et al* (2017)

These dynamics have to incorporate the feedback mechanism that intrinsically links GCMs' default, market turbulence and margin calls on market participants. Furthermore, the fact that for large GCMs central clearing may represent only a fraction of their broader economic activity should be accounted for. Although the system of interconnected CCPs is far too complex to be analysed analytically, it is not beyond reach to study it numerically by developing simulation models that capture the contingent cashflows (including those related to margining and defaults) between all agents. Such models can contribute to better understanding of the following important but complex issues related to the impact of central clearing on over-the-counter (OTC) derivatives portfolios:

- identifying potential systemic risks and contagion introduced by the interconnectedness of the system;

- identifying potential liquidity risks driven by profit and loss (P&L), margin calls, losses due to default and CCP recapitalisation;
- identifying the feedback loop between market volatility and default likelihood;
- identifying the key points of failure;
- identifying the magnitude of sufficiently large scenarios for XYZ Bank to incur a loss or suffer liquidity issues.

The corresponding CCPs and their GCMs for a real-world CCP network are shown in Figure 16.1.

Below, we describe a model that analyses the entire network of CCPs and GCMs. Given the size and intricacy of this network, the model is naturally complex, yet it yields some important insights, both qualitative and quantitative. In particular, it shows that there are material cross-risks between defaults of GCMs and market volatility that must be captured in order to realistically assess default losses and contingent liquidity requirements. Interestingly, the results of our calculations contradict the oft-repeated claim that the move from bilateral clearing to central clearing of OTC derivatives poses a significant threat of contagion through the central counterparties. Our conclusion is predominately due to the fact that the risks attributable to clearing are comparatively small compared with those attributable to the capital held by the diversified financial institutions dominating the CCP landscape, such as XYZ Bank.

In this chapter, which is an extended and updated version of Barker *et al* (2017), we develop a simulation framework to investigate all the risks associated with central clearing mentioned earlier. Additional information can be found in Chapters **??** and 11. Our approach uses the minimum amount of information necessary to analyse the risk of default contagion and liquidity crunches in the CCP framework, but still allows a realistic simulation of possible stressed situations. It is clear that for every GCM that defaults in a specific market scenario, we need to model the loss over IM and DF in each CCP it participates in, and allocate that loss to other GCMs, including XYZ Bank. We also need to model the actual GCM default given a market scenario, and link this to the reduction in the XYZ Bank's capital due to losses on a number of CCPs.

This chapter is organised as follows. In Section 16.1, we present a literature overview. In Section 16.2, we discuss the challenges that have to be addressed in the process of building a theoretically adequate and practically relevant model of central clearing. In Section 16.3, we qualitatively describe various VM, IM and DF calculations performed by CCPs. In Section 16.4, we present the process generating the portfolios of clearing members, given the partial information that a particular GCM possesses. In Section 16.5, we discuss a simple adaptation of the model for Comprehensive Capital Analysis and Review (CCAR) purposes. In Section 16.6, we demonstrate how to simulate the underlying market variables and the feedback mechanism used to generate realistic codependency between volatilities and defaults. We present numerical results in Section 16.7. Finally, we draw some conclusions in Section 16.8.

## 16.1 LITERATURE OVERVIEW

Early models quantifying potential exposure of CCPs can be divided into three main categories:

1. statistical models;
2. optimisation models;
3. option pricing based models.

Statistical models assumed simple underlying dynamics, such as geometric Brownian motion, and derived the probability for the IM to be exceeded within a given time horizon. For instance, Figlewski (1984) calculated the probability of a margin call given a certain percentage of VM and IM.

Optimisation models calculated margins in a way that balances the resilience of CCPs and costs to their members. For example, Fenn and Kupiec (1993) and Baer *et al* (1996) built models along these lines by minimising the total sum of margin, settlement and failure costs.

Option pricing based models explored the fact that the exposure profile of a CCP is approximately equivalent to a combination of call and put options because a GCM can strategically default if the contract loses more value than the posted IM. (This is largely a theoretical possibility.) Day and Lewis (1999) used this framework and estimated prudent margin levels for specific instruments.

When designing its defences, a CCP has to analyse losses conditional on exceeding margins. By its very nature, extreme-value theory (EVT) can be used for this purpose; it has been exploited by several researchers (see, for example, Longin 1999; Broussard 2001). While the use of EVT to set up margins for a single contract is straightforward, it is much more difficult to do this at a portfolio level. Accordingly, CCPs tend not to use EVT directly, relying instead on the intuitive standard portfolio analysis of risk (SPAN) methodology and its variations (see Kupiec 1994). In practice, SPAN has severe limitations when applied to complex portfolios. The value-at-risk-based (VaR-based) IM system, which is better suited for such a task, was discussed by Barone-Adesi *et al* (2002).

More recently, some fundamental topics related to the clearing process have come into focus. For instance, Duffie and Zhu (2011) questioned the premise that central clearing of OTC derivatives can substantially reduce counterparty risk. They argued that some of the expected benefits are lost due to the fragmentation of clearing services, since there is no allowance for interoperability across asset classes and/or CCPs. They argued that the benefit of multilateral netting among many clearing participants across a single class of derivatives over bilateral netting between counterparties across assets depends on the specifics of the clearing process and could be absent in practice.

Arnsdorf (2012) showed that a clearing GCM's CCP risk is given by a sum of exposures to each of the other clearing members, which arises because of the implicit default insurance that each member has provided in the form of mutualised, loss sharing collateral (see also Chapter **??**). He calculated the exposures of GCMs by explicitly modelling the capital structure of a CCP as well as the loss distributions of the individual member portfolios. Arnsdorf assumed that all GCMs are equivalent, which is not the case in practice.

Borovkova and El Mouttalibi (2013) used a network approach to analyse systemic risk in CCPs and showed that the effect of CCPs on the stability of the financial system is rather subtle and not necessarily net positive. They argued that stricter capital requirements have a more beneficial impact on the system than mandatory clearing through CCPs.

Cont and Avellaneda (2013) developed an optimal liquidation strategy for a defaulted GCM portfolio that is based on auctioning parts of the portfolio, unwinding other parts and selling the rest on the market. They modelled an auction with limits on how many positions can be liquidated on a given day due to liquidity considerations, and determined an optimal sale strategy to minimise market risk by using linear programming.

Cumming and Noss (2013) assessed the adequacy of CCPs' default resources and concluded that the best way to model a CCP's exposure to a single GCM in excess of its IM and DF contribution is to use EVT. They drew a simple analogy between the risk faced by a CCP's default fund and that borne by a mezzanine tranche of a collateralised debt obligation (CDO) and used an established framework to model codependency of defaults based on a gamma distribution. Their model is a useful step towards building a proper top-down statistical framework for evaluating the risk of a CCP's member exposures.

Glasserman *et al* (2015) discussed systemic risks in markets cleared by multiple CCPs. The desire to minimise liquidity add-ons creates incentives for swaps dealers to split their positions between multiple CCPs. As a result, potential liquidation costs are hidden from individual CCPs, so that, as a group, they tend to underestimate these costs.

Murphy and Nahai-Williamson (2014) discussed approaches to the analysis of DF adequacy, and analysed various design choices and regulatory constraints for the default waterfall. They concentrated on the "cover 2" requirement because it is a minimum internationally acceptable standard for a CCP. They showed how to use market data to estimate the complete distribution of a CCP's stressed credit risk and studied the prudence of "cover 2" as a function of the number of GCMs.

Elouerkhaoui (2015) developed a method for calculating credit value adjustment (CVA) for CCPs using the Marshall–Olkin correlation model for CDOs and derived the master equations for bilateral CVA, funding value adjustment (FVA) and margin value adjustment (MVA). By its very nature, his approach suffers from the fact that it assumes the defaults of GCMs (and hence of the CCP itself) to be idiosyncratic and hence not directly linked to the behaviour of the underlying cleared product.

Ghamami (2015) introduced a risk measurement framework that coherently specifies all layers of the default waterfall resources of typical derivatives CCPs, and produced a risk sensitive definition of the CCP risk capital.

Armenti and Crepey (2017) challenged the "cover 2" and IM proportional rules for the respective sizing and allocation of the default fund, and instead proposed economic capital specifications based on expected shortfalls of the one-year-horizon P&L of the CCP. They also proposed an IM-raising strategy where the clearing member delegates the posting of its IM to a specialist lender, and showed that this strategy results in a significant MVA compression compared with the classical approach.

Barker *et al* (2017) proposed a model for the credit and liquidity risks faced by GCMs of the network of CCPs. By considering this network in its entirety, they investigated the distribution of losses to default fund contributions and contingent liquidity requirements for each GCM. They concluded that liquidity risks (margin calls) are more dangerous to large diversified GCMs than credit risks.

Berlinger *et al* (2017) analysed the effects of different margin strategies on the loss distribution of a CCP during different crises and found that anti-cyclical margin strategies might be optimal not only for regulators aiming to reduce systemic risk, but also for CCPs focusing on their micro-level financial stability.

Menkveld (2017) emphasised the fact that CCP risk management does not account for risks associated with crowded positions. He proposed an exposure measure based on tail risk in trader portfolios, which identifies and measures crowded risk and assigns it to traders according to the polluter-pays principle.

Lipton (2018) analysed the pros and cons of moving trade execution, clearing and settlement to blockchain and concluded that the advantages of such a move are not as clear-cut as its proponents claim. Still, by using permissioned private ledger(s), costs can potentially be cut and the speed of clearing and settlement somewhat increased while the number of failures can be reduced.

## 16.2 CHALLENGES OF CENTRAL COUNTERPARTY CLEARING HOUSE MODELLING

In order to build a proper model for a CCP, or, even harder, a system of CCPs, a modeller needs to describe the following effects:

- the asymmetry of information due to the fact that a GCM is exposed through its DF to losses on a proportion of the defaulting members' entire portfolio;

- a feedback mechanism that intrinsically links GCM default, market turbulence and liquidity calls on market participants, so that expected loss becomes strongly path dependent;
- the individual nature of different GCMs, varying from large diverse financial institutions where clearing makes up a small amount of their business, to proprietary funds for which a default event will be driven purely by margin calls on cleared trades;
- the individual aspects of each CCP, including their IM and DF methodologies, loss allocation process, assessment and waterfall rights;
- the interconnectedness of the CCPs themselves due to the fact that the default of a major GCM is likely to affect several CCPs simultaneously and may have "knock-on" effects;
- the changes in IM and DF requirements as the system evolves, which are particularly important for liquidity considerations.

The complexity of the model and the need to examine extreme events imply that there is a large computational effort required to use it in practice.

Assuming that (some of) these effects are properly modelled, the following key topics regarding CCP risk can be addressed:

- estimating the total CCP portfolio given the asymmetry of information;
- identifying potential causes of defaults and the corresponding default scenarios;
- analysing market circumstances, given the strong interlink between market volatility and GCMs defaults, which could challenge the stability of the CCP;
- understanding implications of the CCP default for a given GCM;
- evaluating the increase in risk for a specific GCM, given the interconnectedness of a market with multiple CCPs all with the same set of GCMs;
- identifying the benefits (if any) of using multiple CCPs to clear the same trade type, given the interconnectedness of the system;

- finding the expected and 99th-percentile cumulative loss up to a time horizon, $T$, from a GCM's CCP exposures (hits to default funds and possible CCP failures);
- estimating the expected and 99th-percentile cumulative call on liquidity up to a time horizon, $T$;
- estimating the liquid capital a bank would require to survive a large systemic disturbance;
- identifying the indicators that could be used to predict a large disturbance;
- determining which GCM (or set of GCMs) represents the largest systemic risk should it (they) default (including parent versus legal entity);
- identifying the weakest CCP and that with the largest systemic impact should it fail;
- ordering CCPs in terms of risk and systemic impact;
- determining the tipping point in terms of the number of defaults and market moves that would lead from a simple loss scenarios to a systemic failure;
- analysing the impact of various changes to the way CCPs operate on the system as a whole and an individual GCM's risks.

While analysing these topics in their entirety is very difficult, we can make some progress along the lines described below.

## 16.3 MARGIN CALCULATION

To protect themselves against the adverse impact of a GCM default, CCPs set up extensive processes requiring their GCMs to post VMs, IMs and DF contributions in order to cover the mark-to-market (MtM) moves of the GCMs' exposures, as well as any eventual losses, which would be distributed between the defaulter, the survivors and the CCP itself. In order to build a useful model for a given set of market data and a portfolio of trades, we need to design a fast and accurate calculation procedure for VMs, IMs and the DF for the total set of GCM portfolios on a given CCP.

### 16.3.1 Variation margin

First, we represent the state of the market at time $t$ by $\boldsymbol{X}(t) = (X_1(t), \dots, X_n(t))^{\mathrm{T}}$, where $X_i(t)$ corresponds to a financial quantity of importance, such as a par swap rate, spot foreign exchange (FX) rate or credit spread. The actual model for the generation of the corresponding scenarios is described in Section 16.6.

The calculation of the incremental VM called over the time interval $[t_i, t_{i+1}]$ for the portfolio held by the $k$th GCM of the $j$th $\mathrm{CCP}_j$ is straightforward. It is given by the change in its MtM

$$\begin{aligned}\mathrm{VM}^{\mathrm{CCP}_j}_{\mathrm{GCM}_k}(t_{i+1}) - \mathrm{VM}^{\mathrm{CCP}_j}_{\mathrm{GCM}_k}(t_i) \\ = \sum_{\phi \in \Phi^{\mathrm{CCP}_j}_{\mathrm{GCM}_k}} V_\phi(\boldsymbol{X}(t_{i+1}), t_{i+1}) - V_\phi(\boldsymbol{X}(t_i), t_i) \qquad (16.1)\end{aligned}$$

where the summation is over all trades, $\phi$, in the portfolio $\Phi^{\mathrm{CCP}_j}_{\mathrm{GCM}_k}$ that $\mathrm{GCM}_k$ holds with $\mathrm{CCP}_j$ at time $t_i$, and $V_\phi(\boldsymbol{X}(t), t)$ is the value of trade $\phi$ at time $t$ in market state $\boldsymbol{X}(t)$. Below we use the shorthand notation $\mathrm{VM}^j_k$ for $\mathrm{VM}^{\mathrm{CCP}_j}_{\mathrm{GCM}_k}$, and similarly for other quantities of interest. For brevity, we suppress the superscript $j$ when there is no ambiguity.

VMs are symmetric and can be paid by both GCMs and CCPs.

### 16.3.2 Initial margin

The rationale behind the IM charge is that it covers the period between the default and the sell-off of the defaulter's positions. For OTC derivatives this period is typically five days. The Committee on Payment and Settlement Systems–Technical Committee of the International Organization of Securities Commissions (2012, p. 50) articulated the IM requirements as follows:

> a CCP should adopt initial margin models and parameters that are risk-based and generate margin requirements sufficient to cover its potential future exposure to participants in the interval between the last margin collection and the close out of positions following a participant default.

Every CCP has its own margining methodology (see, for example, Chapter 2), but a typical IM calculation is split into a VaR or conditional VaR (CVaR) component and a set of add-ons. The VaR/CVaR component is calculated across the portfolio losses as follows:

(i) a set of scenarios is created by looking at a time series of five days' changes in market data over some historic period;

(ii) these changes are weighted by a multiple of the ratio of current realised volatility to historic realised volatility;

(iii) new scenarios are created by using the current market data and the set of market changes;

(iv) the five-day loss on the portfolio for each scenario is calculated;

(v) the VaR or CVaR components are calculated using these.

Typical add-ons intended to cover various non-linear effects include:

- a liquidity add-on, reflecting the fact that larger portfolios are difficult to off-load in a timely fashion;
- a basis add-on, accounting for tenor basis exposure and similar effects;
- a diversification add-on offsetting diversification benefit on illiquid currencies;
- an unscaled VaR/CVaR floor, which is set at VaR/CVaR calculated without volatility scaling to reduce procyclicality.

IMs are asymmetric and are paid only by GCMs to CCPs.

To accelerate the calculations of $\mathrm{IM}_k^j$ for the $k$th GCM of the $j$th CCP, the VaR/CVaR component is often approximated by using regression against a collection of carefully chosen representative portfolios that are sufficiently small so as to not incur any add-ons, while the add-ons are applied deterministically. Thus, the IM for a given portfolio is calculated as follows

$$\mathrm{IM}_k^j(t) = \mathrm{VaR}(\{\boldsymbol{X}(u)\}_{u\leqslant t}; \boldsymbol{a}(t)) + \mathrm{AddOn}(t) \tag{16.2}$$

where $\boldsymbol{a}(t) = (a_1(t), \dots, a_n(t))^{\mathrm{T}}$ represents the regression coefficients of the portfolio held at time $t$ against the small benchmark portfolios for which IM has been calculated exactly by using information provided by the CCP. Here $\mathrm{VaR}(\{\boldsymbol{X}(u)\}_{u\leqslant t}; \boldsymbol{a}(t))$ denotes the VaR component of the IM for a portfolio represented by the regression coefficients and adjusted for new simulated market data by taking into account the change in the ratio of current market volatilities to historic volatilities and replacing one of the VaR or CVaR elements if the new scenario creates a loss large enough to warrant such a step. The contribution to the volatility of the add-on in Equation 16.2 is thus virtually always ignored and it is frozen at its initial value.

Procyclicality in IM calculation emerges from the following considerations. When markets go into a stressed period, IM requirements increase, so GCMs get liquidity calls at the worst possible time. There are two main sources of procyclicality:

1. an increase in the volatility multiplier accounting for current market volatility relative to its historic level;
2. the introduction of new extreme moves to the tail of the loss distribution.

The second effect is particularly dangerous, since it occurs simultaneously with a large VM call.

Until now, examples of IM failures have been few and far between. We give one such example, which illustrates the above points. In 2010, HQ, a Stockholm-based bank, collapsed due to a large position in strike spread volatility plays on Eurex. The Eurex model did not account for the difference in volatilities across different strike positions for equity options. As a result, HQ had very low IM requirements but ran up equity losses of Skr1.17 million (US$175 million), well above their margin requirements. Plaintiffs commissioned an expert report, which concluded that Eurex did not sufficiently account for the vega and skew risks, and did not charge enough to compensate for the portfolio's high concentration risks (see Clancy 2016).

### 16.3.3 Default fund

The DF is typically based on a set of stressed scenarios including historic and hypothetical market moves applied to current market data. For example, in 2012 the European Commission introduced its European Market Infrastructure Regulation (European Parliament and the Council of the European Union 2012), which require a CCP to have sufficient collateral across its IM and DF to cover the simultaneous defaults of two of its largest GCMs under a range of severe but possible historic and hypothetical market scenarios.

The calculation of the DF can be performed as follows:

(i) for each GCM portfolio, the IM is computed;

(ii) the set of stressed scenarios is applied, and for each scenario any loss on the GCM portfolios is calculated;

(iii) the corresponding margin shortfall is given as (MtM loss + IM) if MtM is negative;

(iv) the two biggest losses are then summed, and the maximum of these sums over all scenarios minus any CCP "skin in the game" (capital) gives the DF;

(v) the relative contribution of each GCM to the DF is computed in proportion to its IM contribution to the total IM.

The total DF for $\text{CCP}_j$ at time $t$, $\text{DF}_j(t)$, is given by a "cover 2" principle

$$\text{DF}_j(t) = \max_{\vartheta \in S_j} \max_{k \neq l} [\text{LOIM}^j_k(t, \vartheta) + \text{LOIM}^j_l(t, \vartheta) - \Upsilon_j]^+ \tag{16.3}$$

where the maximum is taken over all stress scenarios, $S_j$, and distinct pairs of GCMs $(k, l)$, $\Upsilon_j$ is the CCP's "skin in the game" and the losses over initial margins (LOIM) are given by

$$\text{LOIM}^j_\gamma(t, \vartheta) = \left[ \sum_{\phi \in \Phi^j_\gamma(t)} (V_\phi(t, \boldsymbol{X}(t)) - V_\phi(t, \boldsymbol{X}^\vartheta(t))) + \text{IM}^j_\gamma(t) \right]^+ \tag{16.4}$$

where $\gamma = k, l$. Here the summation is preformed over trades $\phi$ in the portfolio $\Phi^j_\gamma(t)$, and $V_\phi(t, \boldsymbol{X}^\vartheta(t))$ and $V_\phi(t, \boldsymbol{X}(t))$ are, respectively, the values of $\phi$ at time $t$ in market state $\boldsymbol{X}(t)$ with and without the stress scenario, $\vartheta$, applied. Although CCPs define a large set of stress scenarios, there are few "binding" ones, so the original set may be replaced with a fairly small subset. We checked this observation experimentally by analysing thousands of actual and randomly generated realistic portfolios and examining which scenarios generated the largest losses. A very high proportion of the scenarios used by the CCP were never binding.

The DF is asymmetric and is paid only by GCMs to CCPs.

To make our simulations as realistic as possible, we used the methods of allocating the DF between the members that were prescribed by the CCPs themselves. To achieve a "cover 2" criterion, CCPs balance IMs and DF amounts. For every dollar decrease in DF there is a need for a considerably larger dollar increase in IM. As a rule of thumb, the ratio is \$1 to \$10. Large IMs and a small DF result in a very low risk of loss to survivors' DFs, but very high margining requirements. In other words, low credit risk implies high liquidity risk. Small IMs and a large DF result in a low risk of large calls on liquidity from day-to-day margining, but significantly greater chances of the need to replenish the DF. Not surprisingly, high credit risk

**Table 16.1** CCPs' provided data

| CCP | CCP provided | | | |
|---|---|---|---|---|
| | CCP IM total | CCP DF | Top 5 IM (%) | Top 10 IM (%) |
| LCH | 115,545,868,500 | 6,524,419,500 | 24.82 | 40.88 |
| CME IRS | 24,837,454,393 | 2,632,833,613 | 68.57 | 91.74 |
| CME Base | 84,570,587,715 | 3,654,030,732 | 51.70 | 76.41 |
| CME CDS | 1,393,981,721 | 650,000,000 | 73.50 | N/A |
| ICE Clear US | 15,808,205,520 | 415,067,300 | 59.37 | 83.06 |
| ICE Clear Europe | 38,211,320,837 | 1,900,000,000 | 36.37 | 54.99 |
| ICE Clear Credit | 24,511,892,147 | 1,972,670,844 | 55.00 | 74.00 |
| OCC | 50,136,632,269 | 7,940,558,697 | 45.00 | 63.00 |
| Nodal Clear | 191,221,324 | 129,903,367 | 75.40 | N/A |
| MGEX | 305,331,089 | 56,355,000 | 88.26 | N/A |
| NGX | 1,740,350,386 | 275,633,878 | 24.70 | 40.30 |
| NSCC | 5,272,000,000 | 834,970,830 | 29.00 | 47.00 |
| FICC–GSD | 11,853,000,000 | 1,877,258,962 | 36.00 | 50.00 |
| FICC–MBSD | 4,437,000,000 | 702,724,881 | 43.00 | 54.00 |
| Total | 378,814,845,901 | 29,566,427,603 | | |

*Source:* central counterparty clearing houses

implies low liquidity risk. When designing its defences, CCPs need to balance credit risk with liquidity considerations.

### 16.3.4 Analysis of CCPs' provided data

Typical total IMs and DF for several representative CCPs are shown in Table 16.1.

In order to process the above information, we postulate that initial margins are exponentially distributed with the rank of GCMs. More specifically, to each CCP, $j$, and GCM, $k \in 1, \dots, K_j$, we assign a rank $\Gamma_k^j \in \{1, \dots, K_j\}$, based upon data sourced from publicly available information, such as financial statements. For a given CCP, we fit a two-parameter exponential distribution to the gross notional of the members, motivated by the analysis of Murphy and Nahai-Williamson (2014)

$$\mathrm{IM}_k = \alpha \mathrm{e}^{-\beta \Gamma_k} \tag{16.5}$$

**Table 16.2** Values of $\beta_5$, $\beta_{10}$ and $\beta^*$, which characterise the exponential distribution of $\mathrm{IM}_k$

| CCP | $\beta_5$ | $\beta_{10}$ | $\beta^*$ |
|---|---|---|---|
| LCH | 0.0571 | 0.0526 | 0.0548 |
| CME IRS | 0.2315 | 0.2494 | 0.2404 |
| CME Base | 0.1455 | 0.1444 | 0.1450 |
| CME CDS | 0.2656 | — | 0.2656 |
| ICE Clear US | 0.1801 | 0.1775 | 0.1788 |
| ICE Clear Europe | 0.0904 | 0.0798 | 0.0851 |
| ICE Clear Credit | 0.1597 | 0.1347 | 0.1472 |
| OCC | 0.1196 | 0.0994 | 0.1095 |
| Nodal Clear | 0.2805 | — | 0.2805 |
| MGEX | 0.4284 | — | 0.4284 |
| NGX | 0.0567 | 0.0516 | 0.0542 |
| NSCC | 0.0685 | 0.0635 | 0.0660 |
| FICC–GSD | 0.0893 | 0.0693 | 0.0793 |
| FICC–MBSD | 0.1124 | 0.0777 | 0.0950 |

*Source:* central counterparty clearing houses

Accordingly, for large $K$

$$\left.\begin{aligned} \mathrm{IM}_{\mathrm{tot}} = \sum_{k=1}^{K} \mathrm{IM}_k &= \frac{\alpha(1-\mathrm{e}^{-K\beta})}{\mathrm{e}^{\beta}-1} \approx \frac{\alpha}{\mathrm{e}^{\beta}-1}, \\ \sum_{\{k \mid 1 \leqslant \Gamma_k \leqslant 5\}} \mathrm{IM}_k &= \frac{\alpha(1-\mathrm{e}^{-5\beta})}{\mathrm{e}^{\beta}-1}, \\ \sum_{\{k \mid 1 \leqslant \Gamma_k \leqslant 10\}} \mathrm{IM}_k &= \frac{\alpha(1-\mathrm{e}^{-10\beta})}{\mathrm{e}^{\beta}-1}. \end{aligned}\right\} \tag{16.6}$$

In order to calibrate $\beta$, two auxiliary exponents, $\beta_5$ and $\beta_{10}$, are computed from the above equations and the proportional allocation to top-five and top-ten IMs, as publicly reported by the CCPs

$$\left.\begin{aligned} \phi_5 &= \frac{\sum_{\{k \mid 1 \leqslant \Gamma_k \leqslant 5\}} \mathrm{IM}_k}{\mathrm{IM}_{\mathrm{tot}}}, & \phi_{10} &= \frac{\sum_{\{k \mid 1 \leqslant \Gamma_k \leqslant 10\}} \mathrm{IM}_k}{\mathrm{IM}_{\mathrm{tot}}} \\ \phi_5 &= (1-\mathrm{e}^{-5\beta_5}), & \beta_5 &= -\frac{\ln(1-\phi_5)}{5} \\ \phi_{10} &= (1-\mathrm{e}^{-10\beta_{10}}), & \beta_{10} &= -\frac{\ln(1-\phi_{10})}{10} \end{aligned}\right\} \tag{16.7}$$

The final exponent, $\beta^*$, is based on the average of $\beta_5$ and $\beta_{10}$

$$\beta^* = \tfrac{1}{2}(\beta_5 + \beta_{10}) \tag{16.8}$$

while $\alpha$ is determined as follows

$$\alpha = \mathrm{IM}_{\mathrm{tot}}(\mathrm{e}^{\beta^*} - 1) \tag{16.9}$$

As shown in Table 16.2, for most CCPs, $\beta_5$ and $\beta_{10}$ are very close; thus, the use of the exponential distribution is justified.

### 16.3.5 The end of the waterfall

What happens at the end of the waterfall is not always sufficiently clearly articulated by CCPs, at times deliberately. In general, if the IM and DF contributions of the defaulter, as well as the DF contributions of survivors and CCP's own "skin in the game", are not enough to alleviate the entire loss due to default, the following steps can be taken:

(i) the DF is reassessed, allowing the CCP to ask for extra contributions to the DF (known as unfunded DF) upon the default of one or more GCMs, which are typically capped at three to four times the original DF;

(ii) a VM gains haircut (VMGH) allows the CCP not to return the full gains on any GCM positions that are in profit, usually capped at half the current DF or a time-cap;

(iii) an IM haircut allows the CCP to use a proportion of surviving members IM to cover the loss;[1]

(iv) a partial tear-up results in the unwinding of all the defaulting members' trades, which may leave surviving GCMs with large unwanted IM/Risk positions;

(v) full tear-up results in the unwind of all cleared trades;

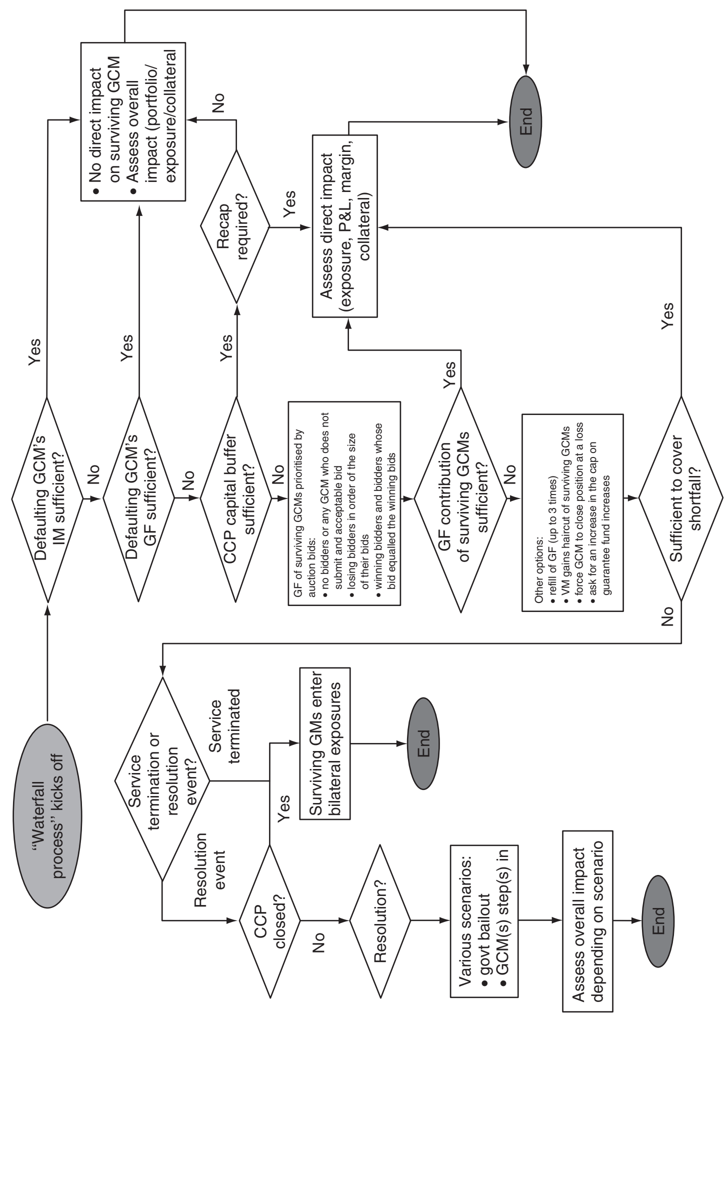
(vi) the CCP unwinds, resulting in all surviving GCMs going back to bilateral arrangements or novating trades to another CCP;

(vii) GCMs step in and take over running of the CCP;

(viii) the government takes responsibility for the CCP.[2]

Possible actions of CCPs and their GCMs are summarised in Figure 16.2.

**Figure 16.2** Default waterfall



## 16.4 GENERAL CLEARING MEMBERS' PORTFOLIOS

Portfolios of typical GCMs consist of numerous long and short trades with slightly different characteristics (eg, maturity and coupon), which partially cancel each other. We refer to the total notional over long and short positions as the "gross" notional, and the difference as the "net" notional. We emphasise that risks are ultimately determined by net positions, so net notionals are of primary concern; however, gross notionals should not be discarded, since they provide useful and important information on accumulated historical volumes.

Under normal circumstances, XYZ Bank deals only with CCPs. However, it becomes potentially exposed to the positions of other members in the event of their default; yet, these positions are purposely obfuscated by CCPs, and hence unknown to GCMs, including XYZ Bank. Typically, CCPs publish gross notionals aggregated across all members for certain categories of derivatives that are discriminated by the type of trade, currency and tenor. A representative example would be the aggregate gross notional for fixed versus six-month floating Euro Interbank Offered Rate swaps for tenors in the range of 2Y to 5Y (alongside other aggregates). We use these aggregates as a measure of the relative scale of the exposures of the CCP to different trade types, currencies and tenors.

To exploit all relevant available information, we align our methodology to the categorisation used by the CCPs when reporting the corresponding aggregate gross notionals. To this end, we fix a particular category, $\pi \in \Pi$, where $\Pi$ is the set of all categories for which the CCP under consideration discloses aggregate gross notionals. We develop a randomisation scheme exploring the space of valid configurations of the unknown positions of other GCM's, satisfying the known constraints, including values related to XYZ Bank's positions and the aggregate gross notionals published by the CCPs. As before, the gross notional for all GCMs and for XYZ Bank can be written in the form

$$\left.\begin{aligned} \sum_{k=1}^{K} \alpha^{\pi} \exp(-\beta^{\pi} \Gamma_k) &= N^{\pi}, \\ \sum_{k \in \varkappa_{\mathrm{XYZ}}} \alpha^{\pi} \exp(-\beta^{\pi} \Gamma_k) &= N^{\pi}_{\mathrm{XYZ}} \end{aligned}\right\} \qquad (16.10)$$

where $\varkappa_{\mathrm{XYZ}}$ is the set of indices of XYZ Bank's members,[3] and $N^{\pi}$ and $N^{\pi}_{\mathrm{XYZ}}$ are the gross notionals for category $\pi$ aggregated over all members and XYZ Bank's members, respectively. System 16.10 of equations for $\alpha^\pi, \beta^\pi$ is easily solved numerically. The corresponding fitted net notional for $k$ is written as

$$N_k^\pi = \alpha^\pi \exp(-\beta^\pi \Gamma_k) \tag{16.11}$$

We wish to generate randomised net notionals, $\{\Delta_k^\pi\}_{k=1}^K$ in such a way that

$$\sum_{k=1}^{K} \Delta_k^\pi = 0 \tag{16.12}$$

so that the CCP is market neutral, and

$$\Delta_k^\pi \in N_k^\pi \mathcal{I}_R, \quad \forall k \notin \varkappa_{\mathrm{XYZ}}, \qquad \Delta_k^\pi = \delta_k^\pi, \quad \forall k \in \varkappa_{\mathrm{XYZ}} \tag{16.13}$$

where $R \in [0, 1]$ is a parameter controlling the relative size of the net and gross positions, $\mathcal{I}_R$ is the interval $[-R, R]$ and $\{\delta_k\}_{k \in \varkappa_{\mathrm{XYZ}}}$ are the known net positions for XYZ Bank's members. Roughly speaking, $R$ is a proportional trading delta limit that, for reasons of parsimony, is assumed to be independent of $k$ and $\pi$. This assumption could be easily removed if necessary.

In order to generate unknown positions $\Delta_k^\pi$, $k \notin \varkappa_{\mathrm{XYZ}}$, in such a way that conditions 16.12 and 16.13 are satisfied, we introduce the negative total sum of all known positions, $\bar{\Delta}^\pi = -\sum_{k \in \varkappa_{\mathrm{XYZ}}} \delta_k^\pi$, define the ratio $r^\pi = \bar{\Delta}^\pi / \sum_{k \notin \varkappa_{\mathrm{XYZ}}} N_k^\pi$ and allocate $\bar{\Delta}^\pi$ proportionally between GCMs with $k \notin \varkappa_{\mathrm{XYZ}}$, $\bar{\Delta}_k^\pi = r^\pi N_k^\pi$. Since none of the clearing members, including XYZ Bank, dominates any particular CCP, we can assume without loss of generality that $|r^\pi| < R$. For each $k \notin \varkappa_{\mathrm{XYZ}}$ we consider the interval $\mathcal{I}_{R-|r^\pi|}^\pi = [-(R - |r^\pi|), (R - |r^\pi|)]$ and generate independent random numbers $u_k^\pi$ uniformly distributed on $\mathcal{I}_{R-|r^\pi|}^\pi$. We define the following quantities

$$U^\pi = \sum_{k \notin \varkappa_{\mathrm{XYZ}}} u_k^\pi N_k^\pi, \qquad V^\pi = \sum_{k \notin \varkappa_{\mathrm{XYZ}}} u_k^\pi N_k^\pi \chi_{u_k^\pi U > 0}, \qquad W^\pi = \frac{U^\pi}{V^\pi} \tag{16.14}$$

where $\chi$ is the indicator function. Since $u_k^\pi$ possesses a density, it is clear that $V^\pi \neq 0$ almost surely, so that $W^\pi$ is well defined, and $0 < W^\pi \leqslant 1$. Finally, we define the net position of the $k$th GCM as follows

$$\Delta_k^\pi = \bar{\Delta}_k^\pi + u_k^\pi (1 - W^\pi \chi_{u_k^\pi U^\pi > 0}) N_k^\pi = (r^\pi + u_k^\pi (1 - W^\pi \chi_{u_k^\pi U^\pi > 0})) N_k^\pi \tag{16.15}$$

Our algorithm implies that we proportionally reduce positions for GCMs with $u_k^\pi$ having the same sign as $U^\pi$, and keep positions for other GCMs fixed. A simple calculation yields

$$\sum_{k \notin \mathcal{K}_{\mathrm{XYZ}}} \Delta_k^\pi = \bar{\Delta}^\pi + U^\pi - \frac{U^\pi}{V^\pi} V^\pi = \bar{\Delta}^\pi \tag{16.16}$$

so that condition 16.12 is satisfied, and

$$\begin{aligned} |\Delta_k^\pi| &= |\Delta_k^\pi - \bar{\Delta}_k^\pi + \bar{\Delta}_k^\pi| \\ &\leqslant |\Delta_k^\pi - \bar{\Delta}_k^\pi| + |\bar{\Delta}_k^\pi| \\ &\leqslant |u_k| + |r^\pi| N_k^\pi \leqslant (R - |r^\pi| + |r^\pi|) N_k^\pi \\ &= R N_k^\pi, \end{aligned} \tag{16.17}$$

so that condition 16.13 is also satisfied.

## 16.5 COMPREHENSIVE CAPITAL ANALYSIS AND REVIEW STRESS ANALYSIS FOR CENTRAL COUNTERPARTY CLEARING HOUSES

For a lot of CCAR-related analysis, we need only to look at individual CCPs in isolation and calculate the expected loss for XYZ Bank given a set of GCM defaults and a pre-specified market shock. To do this we

(i) construct a set of scenarios that includes the Federal Reserve adverse and severely adverse scenarios,

(ii) calculate the present value (PV) with current and stressed market data for each portfolio of the defaulting GCMs,

(iii) calculate total losses via the difference in the PV with current market data (representing default time) and the PV with shocked data (representing the market at time of resolution), since the VM covers any losses before the default,

(iv) sum all losses for defaulted GCMs,

(v) run through the waterfall process, calculating all losses over the defaulter's IM, the DF and the CCP's "skin in the game",

(vi) calculate XYZ Bank's portion of the losses,

(vii) repeat the procedure for a number of randomly drawn portfolio configurations and calculate the expected loss given by the average of these results.

One interesting alternative approach is to model LOIM by using the CCP's own estimates, in the spirit of Arnsdorf (2012) and Chapter **??** of this volume. In this approach, a typical CCP uses a "cover 2" framework for modelling DF, and loss given default for each GCM, $k$, is considered proportional to its $\mathrm{IM}_k$ contribution. Thus, loss to the DF caused by the default of the largest GCM is $\frac{1}{2}\mathrm{DF}_{\mathrm{tot}}$. A fractional contribution to the total IM pool for the $k$th GCM, $\omega_k$, is defined as follows

$$\omega_k = \frac{\mathrm{IM}_k}{\mathrm{IM}_{\mathrm{tot}}} = \exp(-\beta^* \mathit{\Gamma}_k)(\mathrm{e}^{\beta^*} - 1) \tag{16.18}$$

In the case of default, the loss given default (LGD) of the $k$th GCM is approximated as a fraction of $\frac{1}{2}\mathrm{DF}$, which is associated with the loss of the biggest GCM. Accordingly

$$\mathrm{LGD}_k = \frac{\mathrm{DF}}{2} \frac{\omega_k}{\omega_{\{k \mid \mathit{\Gamma}_k = 1\}}} = \tfrac{1}{2}\mathrm{DF}\exp(-\beta^*(\mathit{\Gamma}_k - 1)) \tag{16.19}$$

The fraction of loss attributable to XYZ Bank is proportional to its fraction of the total IM pool

$$\mathrm{LGD}_{k,\mathrm{XYZ}} = \frac{\mathrm{DF}}{2}\exp(-\beta^*(\mathit{\Gamma}_k - 1))\frac{\mathrm{IM}_{\mathrm{XYZ}}}{\mathrm{IM}_{\mathrm{tot}}} \tag{16.20}$$

This expression is particularly useful, since it does not require knowledge of the actual rank of the XYZ Bank.

Since, in the approach under consideration, defaults and losses are independent, the expected losses to XYZ Bank can be written in the form

$$\mathrm{loss}_{\mathrm{XYZ}} = \sum_{k \notin \varkappa_{\mathrm{XYZ}}} \mathrm{LGD}_{k,\mathrm{XYZ}} \sum_{\varpi \in \Omega_k} p_{\varpi} \tag{16.21}$$

where $\Omega_k$ is the set of all subsets of GCMs containing the $k$th GCM, $\varpi$ is a subset and $p_{\varpi}$ is the probability that all GCMs from this subset default while all other GCMs survive. The probability $p_{\varpi}$ has to be calculated by using a copula-based approach.

Since the above derivation is based on very conservative assumptions, it produces a very rough (but still useful) upper bound for the expected loss.

## 16.6 SCENARIO GENERATION

### 16.6.1 Motivation

In this section, we build a model for the underlying market the model is used later in Section 16.7 to describe the contingent cashflows in the system of GCMs whose initial positions are generated according to the scheme presented in Section 16.4. Our model has to be sufficiently rich in order to support jumps, describing large changes in the market variables on short timescales, including systemic jumps affecting all market variables simultaneously; it also has to account for the fact that periods of high default rates, such as the global financial crisis, are accompanied by high market volatility. To accommodate these requirements, we develop a regime-switching model with regimes driven by the number and size of realised defaults.

### 16.6.2 Regime-dependent drivers

To reflect the fact that the GCMs of interest have different sizes, we introduce weights $w_k > 0$, representing the financial significance of the $k$th GCM, $k = 1, \dots, K$, to the ecosystem. Specifically, we assume that $w_k$ are proportional to the balance-sheet assets of the $k$th GCM and normalise it in such a way that $\sum_k w_k = 1$.

We use $w_k$ to build a stress indicator, $\varXi_t$, of the form

$$\varXi_t = \sum_k w_k \exp(-\kappa(t - \tau_k))\chi_{\tau_k < t} \tag{16.22}$$

where $\chi.$ is an indicator function, $\tau_k$ is the default time of $\mathrm{GCM}_k$ and $\kappa$ represents a rate of mean reversion from a stress state to equilibrium and is set to 1 in the following. The indicator $\varXi_t$, representing the materiality-weighted defaults prior to time $t$, is sandwiched between 0 and 1, ie, $0 \leqslant \varXi_t \leqslant 1$.

We introduce thresholds $0 < m_1 < m_2 < \cdots < m_S = 1$, and define the integer-valued stress state process, $\xi_t^{\boldsymbol{m}}$ by

$$\xi_t^{\boldsymbol{m}} = \begin{cases} 1, & \varXi_t \leqslant m_1 \\ i, & m_{i-1} < \varXi_t \leqslant m_i, \quad i \geqslant 2 \end{cases} \tag{16.23}$$

In each of the $S$ stress states, we use volatility multipliers $1 = \varLambda_1 < \cdots < \varLambda_S$. In practice, choosing $S = 2$ is often sufficient.

We define a regime-dependent Wiener driver, $W_t^{\xi^m}$ with $\xi_t^{\boldsymbol{m}}$-dependent volatilities $\varLambda_{\xi_t^m}$

$$\mathrm{d}\langle W^{\xi^m}, W^{\xi^m}\rangle_t = \varLambda^2_{\xi_t^m}\,\mathrm{d}t \tag{16.24}$$

We also define a regime-switching compound Poisson driver, $N_t^{\xi^m}$, with $\xi_t^{\boldsymbol{m}}$-dependent intensities $\lambda\varLambda_{\xi_t^m}$, and the jump distribution of the form $e^Z - 1$, where $Z \sim N(\mu, \sigma)$ (see, for example, Inglis *et al* 2008). We assume that $N_t^{\xi^m}$ is compensated, so that it is a martingale

$$\mathbb{E}[N_u^{\xi^m} - N_t^{\xi^m} \mid \mathcal{F}_t] = 0 \tag{16.25}$$

for $u \geqslant t$. We use these drivers below to describe the dynamics of the relevant market variables.

Since losses on default and liquidity drains are primarily driven by increments in the value of portfolios over short time horizons, we can neglect measure-dependent drifts and second-order convexity adjustments. In the spirit of Lipton and Sepp (2009), we assume that all processes have a sensitivity to a common regime-dependent Poisson process that we denote by $N_t^{\text{sys},\xi^m}$.

### 16.6.3 Rates process

We assume that interest rates in the $i$th economy have a simple Hull–White-inspired dynamics

$$\mathrm{d}r_t^i = \mathrm{d}\phi_t^i + \mathrm{d}X_t^1 + \mathrm{d}X_t^2 + \beta_i r_{t-}^i \, \mathrm{d}N_t^{\text{sys},\xi^m} + r_{t-}^i \, \mathrm{d}N_t^{i,\xi^m} \tag{16.26}$$

where $\phi_t^i$ is deterministic and used to fit the initial term structure; $X_t^1$ and $X_t^2$ are Ornstein–Uhlenbeck processes driven by regime-dependent correlated Wiener processes $W_t^{1,\xi^m}$ and $W_t^{2,\xi^m}$, which are used to control the relative volatility of rates of different tenors and intracurve spread volatilities, while compound, compensated, regime-dependent Poisson processes $N_t^{\text{sys},\xi^m}$ and $N_t^{i,\xi^m}$ represent systemic and idiosyncratic jumps, respectively; $\beta_i$ is the sensitivity of rates to the systemic jump process; and $t-$ denotes left-hand limit. In the spirit of our analysis, below we calibrate the corresponding parameters to historical, rather than implied, market data (see Section 16.7.2).

For simplicity and expediency, we calculate market observables, such as swap rates and LIBORs, from the state $(X_t^1, X_t^2)$ by applying the functional forms for the corresponding (affine) two-factor Hull–White model (without feedback loop and jump terms).

The model given by Equation 16.26 is a minimally complex model incorporating the following features:

- jumps describing extremal market moves over short time-periods;
- regime-dependent volatilities and intensities capturing the naturally increasing codependency with defaults;

- intracurve spread volatility, producing a reasonable P&L distribution for delta-neutral steepener/flattener positions.

### 16.6.4 FX process

The spot FX rate between the $i$th and $j$th economies is modelled analogously to Equation 16.26

$$\mathrm{d}X_t^{i,j} = \sigma_t^{i,j} X_t^{i,j}\,\mathrm{d}W_t^{i,j,\xi^m} + \beta_{i,j} X_{t-}^{i,j}\,\mathrm{d}N_t^{\mathrm{sys},\xi^m} + X_{t-}^{i,j}\,\mathrm{d}N_t^{i,j,\xi^m} \tag{16.27}$$

where $\sigma_t^{i,j}$ is a deterministic function of time, $\beta_{i,j}$ is the sensitivity to the systemic jump process $N_t^{\mathrm{sys},\xi^m}$ and $N_t^{i,j,\xi^m}$ is an idiosyncratic jump process independent of all else conditional on $\xi^m$.

### 16.6.5 Non-CCP asset process

The non-clearing related assets of the $k$th GCM are driven by a process of the same form, ie

$$\mathrm{d}A_t^k = \sigma_t^k A_t^k\,\mathrm{d}W_t^{k,\xi^m} + \beta_k A_{t-}^k\,\mathrm{d}N_t^{\mathrm{sys},\xi^m} + A_{t-}^k\,\mathrm{d}N_t^{k,\xi^m} \tag{16.28}$$

where $\sigma_t^k$ is a deterministic function of time, $\beta_k$ is the sensitivity to the systemic jump process $N_t^{\mathrm{sys},\xi^m}$ and $N_t^{i,j,\xi^m}$ is an idiosyncratic jump process conditionally independent of $\xi^m$.

### 16.6.6 Default events

We use the Merton–Black–Cox structural model to describe the default of the $k$th GCM (see, for example, Lipton and Sepp 2009). More precisely, its default time is the hitting time of the total position of CCP-related and non-CCP-related activities

$$\tau_k = \inf\{t > 0\colon C_t^k - C_0^k + A_t^k \leqslant B_t^k\} \tag{16.29}$$

where $B_t^k$ is a deterministic barrier, $A_t^k$ is the value of non-CCP-related assets, such as loans and mortgages, and $C_t^k - C_0^k$ is the net cashflow due to CCP-related activities. The barriers are calibrated numerically to target default probabilities for the GCM in question.

Depending on the business model of a particular member, the relative significance of the volatilities attributable to CCP-related activity, $C_t$, and non-CCP-related activity, $A_t$, may vary considerably. To capture this fact, we categorise members as follows:

- large diversified financial institutions, whose assets' volatility is dominated by non-CCP-related activities, $A_t$;

- large markets-driven houses, whose trading books make up a significant part of their business so the volatility of $C_t$ is similar to that of $A_t$;
- trading houses, for which the volatility of $C_t$ is larger than $A_t$.

Assuming that the $k$th GCM of the $j$th CCP has defaulted, the total loss for the $j$th CCP has to be calculated and distributed across the remaining GCMs, including XYZ Bank, via the standard waterfall process. In reality, as mentioned earlier, the waterfall processes vary considerably between CCPs and depend on the outcome of the idiosyncratic auctions, which are hard to model accurately. Instead of burdening ourselves with tasks that cannot be accomplished, we simply assume the DFs' losses are mutualised and distributed between the surviving GCMs in proportion to their IMs. Likewise, the net positions of the defaulter are redistributed to all surviving members proportionally to the size of their IM.

For completeness, we need to model the situation described in Section 16.3.5, where the losses exceed all the CCP's resources and it defaults (although this eventuality is unlikely). In this case we assume that surviving GCMs will provide the VM of cleared trades up to the default time of the CCP; after that, all cleared trades will be unwound at par. The resulting losses will be divided proportionally to the surviving GCM's closing IMs. As mentioned earlier, there has never been a major CCP default; thus, the proposed resolution mechanism may or may not not be completely realistic. Still, it can be argued that this mechanism is reasonable and parsimonious.

In view of the above, the incremental cashflows can be represented as follows

$$C_t^k - C_0^k = -\sum_j \mathrm{IM}_k^j(t) - \mathrm{IM}_k^j(0) + \sum_j \mathrm{VM}_k^j(t) - \mathrm{VM}_k^j(0) - \sum_j \sum_{t_{i+1}<t} \mathrm{loss}_{\mathrm{IMDF}_k^j}(t_i, t_{i+1}) \quad (16.30)$$

where the summation is over all CCPs, $\mathrm{IM}_k^j(t)$ and $\mathrm{VM}_k^j(t)$ are the IM and VM margins, $\mathrm{loss}_{\mathrm{IMDF}_k^j}(t_i, t_{i+1})$ is the loss over IM and DF for $\mathrm{CCP}_j$ allocated to $\mathrm{GCM}_k$ over time interval $(t_i, t_{i+1}]$. This loss is given by

$$\mathrm{loss}_{\mathrm{IMDF}_k^j}(t_i, t_{i+1}) = \frac{\chi_{\tau_k>t}\mathrm{IM}_k^j}{\sum_l \chi_{\tau_l>t}\mathrm{IM}_k^j} \mathrm{loss}_{IMDF^j}(t_i, t_{i+1}) \quad (16.31)$$

with the total loss over IM and DF for $\text{CCP}_j$ given by

$$\text{loss}_{IMDF^j}(t_i, t_{i+1}) = \sum_{l:\tau_l \in (t_i, t_{i+1}]} \Bigg( \sum_{\phi \in \Phi_l^j(t_i)} (V_\phi(t_{i+1}) - V_\phi(t_i)) + \text{IM}_l^j(t_i) + \text{DF}_l^j(t_i) \Bigg)^- \tag{16.32}$$

with the summation over GCMs that have defaulted (if any) in the time interval $(t_i, t_{i+1}]$. The length, $t_{i+1} - t_i$, of the simulation time intervals equals the time horizon corresponding to the VaR methodology of the CCP under consideration (typically five business days).

## 16.7 RESULTS

### 16.7.1 Model simulation flow

For the reader's convenience we describe the simulation steps needed to run the model:

(i) calibrate the GCM weights, initial non-cleared asset levels and default barriers;

(ii) calibrate market dynamic variables;

(iii) compute the GCM portfolios on each CCP for each cleared asset class;

(iv) calculate the initial levels of IM, DF and portfolio values;

(v) simulate one period of market/economic data and GCM non-cleared asset values;

(vi) calculate changes in VM, IM, DF over that period, including DF reassessments;

(vii) calculate any GCM defaults over that period;

(viii) distribute any losses through the CCP waterfalls;

(ix) redistribute defaulted CCPs' portfolios to other GCMs;

(x) resolve any CCP defaults;

(xi) repeat until the end of the time horizon or a full cycle with no defaults, whichever is the later;

(xii) run for desired number of paths;

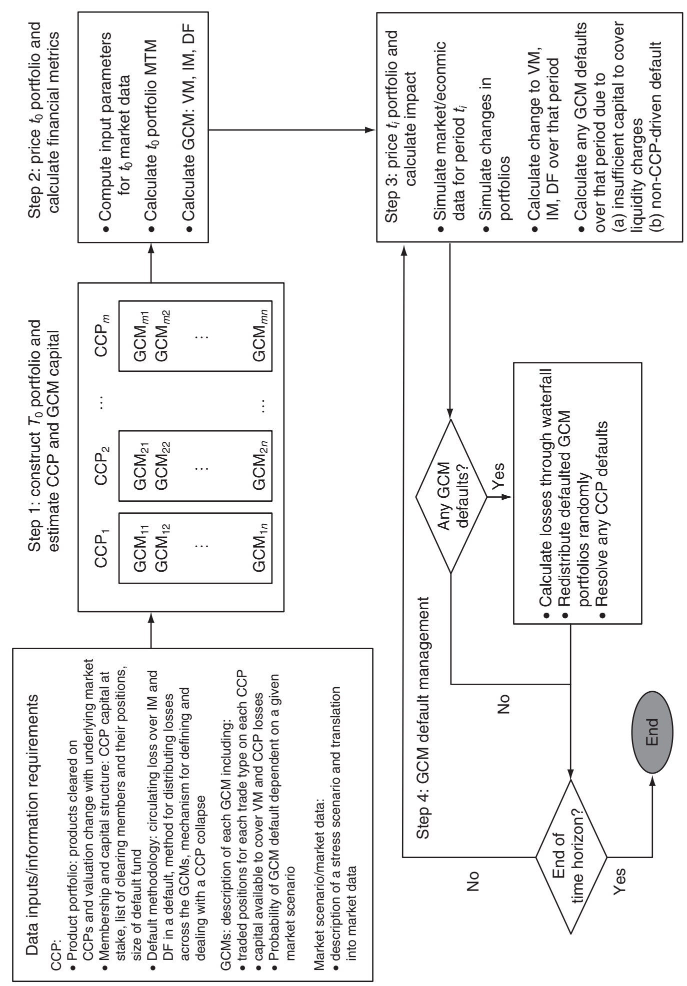



**Figure 16.3** Model flow

(xiii) record statistics for market variables, number and timing of defaults, stress indicator, loss through DF, liquidity requirements, etc;

(xiv) display data either along simulated paths or as distributions at a given time horizon, $T$.

The model flow is illustrated in Figure 16.3.

### 16.7.2 Model calibration

We now use a realistic configuration of the model to calculate the risks faced by XYZ Bank proxying one of the "big four" US banks in scale, with suitably anonymised positions. We consider LCH.SwapClear and the Chicago Mercantile Exchange for US dollar and euro fixed-float swaps and assume that there are 101 clearing members in total, each belonging to both CCPs.

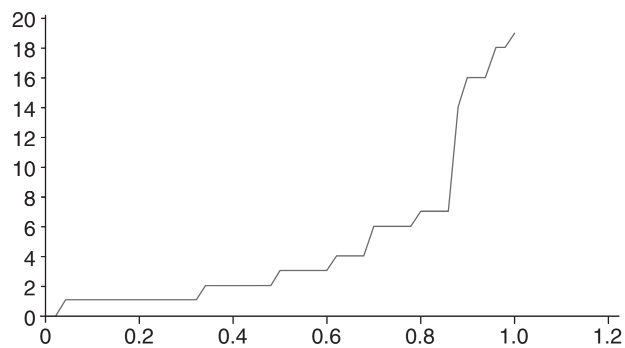
The complexity of the CCP network itself requires that any realistic model aimed at obtaining quantitative rather than qualitative results be complex, large and hence hard to calibrate. Since the model described above is both realistic in terms of its dynamics and robust in terms of the choices made, we feel confident of the legitimacy of any derived results. There are several parts of the model requiring calibration, which we perform in the most conservative way possible. Further details of the techniques used to calibrate the various submodels are given in Barker *et al* (2016).

In our framework, the market is driven by regime-switching sets of jump–diffusion processes. To be specific, we assume there are two states that are differentiated by a volatility multiplier. This volatility multiplier is conservatively set to 2, reflecting the fact that the during the global financial crisis the volatility for rates processes increased by 1.5 times. The underlying processes are calibrated to the current 2Y and 10Y swap rates and their historic volatility. This is done in three stages:

1. the dynamics are calibrated analytically without jumps;
2. a fixed-point iteration is used to calibrate the model, excluding the feedback mechanism described in Section 16.6;
3. the minimal entropy technique is used to calibrate the full market dynamics to the required targets.

The model requires knowing the initial levels of non-CCP assets for GCMs. For the largest GCMs, like XYZ Bank, we use the most

**Figure 16.4** Number of GCM defaults as a function of time

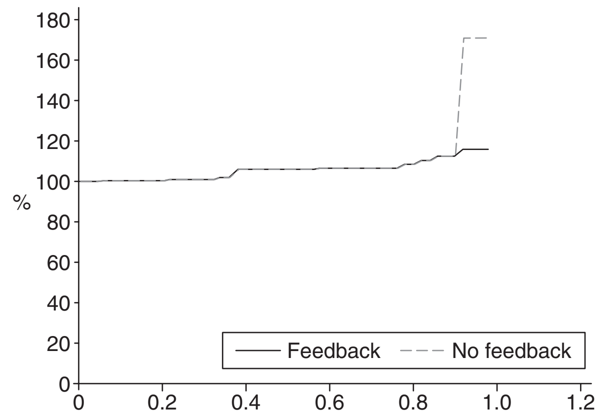



recent published financial statements to fit the initial levels of these assets. Although the corresponding financial information gives only a snapshot in time, we feel that trying to use market observable assets such as credit default swaps would just muddy the waters, add unnecessary complexity and not give any additional genuine insights. We assume that all mid-size GCMs are similar, and use the levels for one of them as a proxy for the rest. This assumption is justifiable due to the fact that the largest GCMs contribute most to the stability of the system as a whole, and have to be modelled as accurately as possible, while smaller GCMs play second fiddle.

In a similar way to the capital asset pricing model, non-CCP assets of each GCM are driven by a common factor, calibrated to the volatility of a portfolio of financial institutions, and an idiosyncratic factor. The GCMs' cleared portfolios are generated using the method described in Section 16.4 with the parameter $R$ in Equation 16.13 chosen in such a way that the total IM and DF reported by the various CCPs, and hence the DF contributions of any known GCM, are reproduced as accurately as possible. Since, for known portfolios, we apply the actual IM calculation methods employed by the various CCPs, we are guaranteed to get the correct IMs. We calibrate the default barriers introduced in Section 16.6.6 to the implied default probabilities of GCMs of interest.

In Figure 16.4, we show the total number of defaults among 101 GCMs over a one-year time horizon for a particular simulation path. This scenario is rather extreme and clearly illustrates the strong adverse impact of the feedback loop on the survival probability

**Figure 16.5** Liquidity ratio as a function of time




of individual GCMs, caused by the increase in default intensities proportional to the realised number of defaults.

In Figure 16.5, we plot the ratio of IM to initial IM against time, with and without a feedback loop. It is clear that feedback effects have a large adverse effect on the liquidity position of a GCM, in agreement with the previous observation.

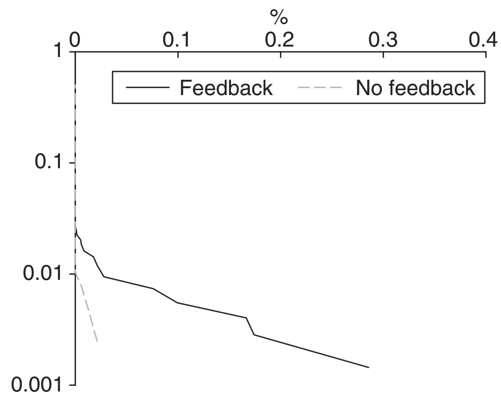
### 16.7.3 Results for a realistic configuration of the model

For a fixed one-year time horizon, we are interested in two random variables $\eta_C$, $\eta_L$ representing the credit losses due to defaults (of other GCMs and CCPs) and potential liquidity drains for XYZ Bank, respectively. These variables are scaled by the shareholder equity of XYZ Bank, and hence represent the relative significance of losses and margin calls to the capital buffer, which is set to US$200 billion, approximating the size of the shareholder equity of a “big-four” US bank. We wish to calculate the complementary cumulative distribution functions (CCDFs) for $\eta_C$ and $\eta_L$ and analyse the qualitative impact of the feedback loop on these distributions.

In order to compute $\mathrm{CCDF}_C$ and $\mathrm{CCDF}_L$ properly, we have to take into account the contingent cashflows between all agents in the CCP network. We calculate these distributions under different scenarios:

- defaults with the feedback-based regime-switching (labelled ‘feedback’);
- defaults only (labelled “default only”);

**Figure 16.6** Logarithmic plot of $\mathrm{CCDF_C}$ at a time horizon of one year for different model configurations




*Source:* Barker *et al* (2017)

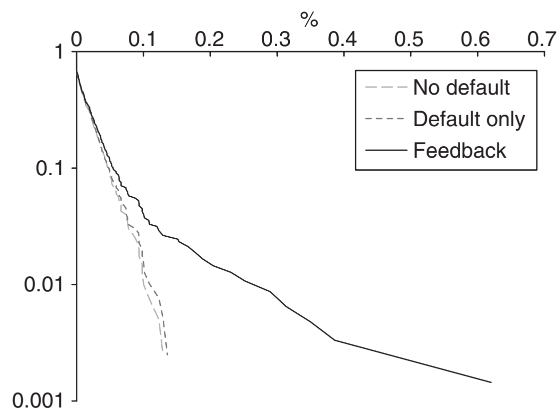
- no defaults.

To ensure comparability of the results across configurations, we hold the expected stress indicator, $\mathbb{E}[\Xi_1]$, fixed as we change the settings of the feedback mechanism. To this end, we use a minimal entropy path-reweighting algorithm (see, for example, Avellaneda *et al* 2001). $\mathrm{CCDF_C}$ and $\mathrm{CCDF_L}$ are plotted with a logarithmic scale on the $y$-axis so that, for example, a $y$-value of 0.01 corresponds to a 99% quantile of the corresponding distributions.

In Figure 16.6, we plot the simulated distribution $\mathrm{CCDF_C}$ for the ratio of the losses due to default (across all of GCMs and CCPs) to shareholder equity of XYZ Bank. This figure demonstrates two key points:

1. the effect of feedback correctly captures the natural wrong-way risk between defaults and market volatility and dramatically amplifies the tail of the loss distribution due to defaults;
2. even making conservative assumptions about the relationship between defaults and market volatility and taking into account the interconnected and complex relationships between the agents of the CCP network, we cannot generate losses large enough to threaten the survival of a well-diversified and well-capitalised financial institution.

**Figure 16.7** Logarithmic plot of $CCDF_L$ at a time horizon of one year for different model configurations




*Source:* Barker *et al* (2017)

In Figure 16.7, we plot the simulated distribution $CCDF_C$ for the ratio of the additional aggregate IM to the shareholder equity of XYZ Bank that it needs to post to the two CCPs in the system. Changes in IM capture the effect of new extreme events entering the VaR look-back period and potential increases in portfolio size due to the absorption of defaulting GCMs' portfolios by surviving GCMs. This figure demonstrates the importance of the likely increases in volatility in stressed market conditions. It also shows, by comparison with Figure 16.6, that, in dollar terms, liquidity drains due to margin calls are significantly larger than losses due to default.

## 16.8 CONCLUSIONS

Regulatory changes since the 2007–10 global financial crisis have resulted in a significant increase in the volume of centrally cleared financial instruments. Yet, the risks associated with central clearing are relatively poorly understood, not least because of the technical difficulties in building a suitable theoretical framework. Such an undertaking requires the ability to describe a large and very intricate network of GCMs and CCPs (see Figure 16.1), and to perform complex calculations related to scenario generation and estimation of the VM, IM, DF and loss waterfall.

We used suitable heuristics to simulate the realistic contingent cashflows between all the agents in the CCP network and to estimate the associated risks. Our model is capable of capturing the substantial wrong-way risk between the volatility of market variables and defaults. Our results indicate that the tail losses and increased liquidity requirements are very real, and, moreover, that liquidity requirements (margin calls) dominate those related to credit risk. The oft-mentioned fear that the wider application of central clearing to OTC derivatives has a destabilising impact on the financial ecosystem as a whole, due to contagion effects transmitted through the CCP network, is not supported by our numerical experiments. While this result is at odds with received wisdom, it can be explained by the fact that losses due to default are a small fraction of the Tier 1 common equity of the diversified financial institutions that dominate the CCP membership. Our recommendation to GCMs assessing the risks and costs associated with their central clearing activities is to focus primarily on funding and liquidity requirements, which, if not provisioned for, can negatively affect business as usual. Still, by their very nature, any CCP-related losses are likely to be realised precisely under extreme market dislocations, making their absorption particularly painful to the GCMs of the CCP network.

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